cube 201-8

CITY OF LIVERPOOL

EDUCATION COMMITTEE

NAME J. Allday.

FORM or CLASS L6 Sc(1).

SUBJECT Pure Maths.

A).

1). 8.691.

n =

2). 9.244.

3). 10.463.

B

1). 8.742.

2). 9.719.

n =

3). 12.170.

$$I = \int_0^1 \frac{x^4 (1-x)^4}{1+x^2}\, dx \qquad x = \tan u \quad dx = \sec^2 x \, du$$

$$= \int_0^1 \frac{\tan^4 x \,(1-\tan x)^4}{\cancel{\sec^2 x}} \cancel{\sec^2 x}\, du$$

$$= \int_0^{\pi/4} \tan^4 u \,(1-\tan u)^4 \, du$$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$= \int_0^{\pi/4} \tan^4 u \,(1 - 4\tan u + 6\tan^2 u - 4\tan^3 u + \tan^4 u)\, du$$

$$= \int_0^{\pi/4} \tan^4 u - 4\tan^5 u + 6\tan^6 u - 4\tan^7 u + \tan^8 u \, du.$$

~~1 - Tan u = p~~

~~dp~~

$$I = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$$

$$= \int_0^1 \frac{x^4(1 - 4x + 6x^2 \cancel{x} - 4x^3 + x^4)}{1+x^2}\,dx$$

$$= \int_0^1 \frac{x^4 - 4x^5 + 6x^6 - 4x^7 + x^8}{1+x^2}\,dx$$

$$\cancel{x^4} + \cancel{x^6} - 4\cancel{x^5} - 4\cancel{x^7} + \cancel{x^6} + \cancel{x^8} + 4x^6$$

$$= \int_0^1 \frac{x^4(1+x^2) - 4x^5(1+x^2) + x^6(1+x^2) + 4x^6}{1+x^2}$$

$$= \int_0^1 x^4 - 4x^5 + x^6 + \frac{4x^6}{1+x^2}\,dx$$

$$= \int_0^1 x^4 - 4x^5 + x^6\,dx + \int_0^1 \frac{4x^6}{1+x^2}\,dx$$

$$I = \int_0^1 \frac{4x^6}{1+x^2}\,dx \qquad \begin{aligned} x &= \operatorname{Tan} u \\ dx &= \sec^2 u\,du \end{aligned}$$

$$= \int_0^{\pi/4} 4\operatorname{Tan}^6 u \; du$$

$$= 4\int_0^{\pi/4} \operatorname{Tan}^6 u\,du = \int_0^{\pi/4} \operatorname{Tan}^4 u\,(1-\sec^2 u)\,du$$

$$= 4\left[\int_0^{\pi/4} \operatorname{Tan}^4 u\,du - \int_0^{\pi/4} \operatorname{Tan}^4 u \sec^2 u\,du\right]$$

$$\begin{aligned} p &= \operatorname{Tan} u \\ dp &= \sec^2 u\,du \end{aligned}$$

$$= \int_0^{\pi/4} \operatorname{Tan}^4 u\,du - \int_0^{1} p^4\,dp.$$

$$= \int_0^{\pi/4} \operatorname{Tan}^2 u\,(1-\sec^2 u)\,du - \int_0^1 p^4\,dp$$

$$= \int_0^{\pi/4} \operatorname{Tan}^2 u\,du - \int_0^1 p^2\,dp - \int_0^1 p^4\,dp.$$

$$= \pi/4 - \int_0^{\pi/4} \sec^2 u\,du - \int_0^1 p^4 - \int_0^1 p^4\,dp.$$

$$4x = \left[\frac{\pi}{4} - 1 - 1/3 - 1/5\right] - 2$$

$$\cancel{= \frac{\pi}{4} - \frac{(15+5+3+30)}{15}}$$

$$\cancel{= \pi\ \frac{\pi}{4} - \frac{53}{15}}.$$

$$\pi - \left(\frac{15+5+3}{15}\right)\times 4 - 2$$

$$\pi - \frac{23\times 4}{15} - 2$$

$$(1+x)^n = a_0 + a_1 x + \cdots + a_n x^n$$

$$\sum_{r=0}^{n} \frac{(-1)^r a_r}{(r+1)} \qquad (-1).$$

$$= \cancel{tha} \; \frac{a_0}{} - \frac{a_1}{2} + \frac{a_2}{3} - \frac{a_4}{4} + \frac{a_5}{5}$$

$$S_n = a_0 - \frac{a_1}{2} + \frac{a_2}{3} - \frac{a_4}{5} + \cdots + \frac{(-1)^r a_r}{(r+1)}.$$

$$(1+x)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n.$$

$(x=1)$.

$$\cancel{2^n = a_0 + a_1 + a_2 + a_3 + \cdots + a_n}$$

$$S_n = a_0 - \frac{a_1}{2} + \frac{a_2}{3} - \frac{a_3}{4} + \cdots + \frac{(-1)^r a_r}{r+1}$$

$$0 = a_0 - a_1 + a_2 - a_3 + \cdots + a_n(-1)^n.$$

$$S_n = 2a_0 - \frac{3a_1}{2} + \frac{4a_2}{3} - \frac{5a_3}{4} + \cdots$$

$$(1+x)^n = a_0 + a_1 x + \dots + a_n x^n.$$

$$\sum_{r=0}^{n} \frac{(-1)^r a_r}{r+1} = a_0 - \frac{a_1}{2} + \frac{a_2}{3} + \dots + \frac{(-1)^n a_n}{(n+1)}.$$

$x = -1$

$-a_3$

$$0 = a_0 - a_1 + a_2 + \dots + a_n(-1)^n$$

$$S_n = a_0 - \frac{a_1}{2} + \frac{a_2}{3} + \dots + \frac{(-1)^n a_n}{n+1}.$$

$-\frac{a_3}{4}$

$$\cancel{S_n = (a_0 + a_1 + a_2 + \dots + a_n)}$$

$x =$

$-2/3\, a_2$

$$0 + 1/2\, a_1 + \cancel{a_2} + 3/4\, a_3 = S_n$$

$$S_n = \frac{a_1}{2} - \frac{2}{3} a_2 + \frac{3}{4} a_3 - \frac{4}{5} a_4 + \dots$$

$$\sum_{r=0}^{n} \frac{(-1)^r a_r}{r+1} = a_0 - \frac{a_1}{2} + \frac{a_2}{3} + \cdots + \frac{(-1)^n a_n}{(n+1)}$$

$$\cancel{\log_e (1+p) = p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{x^4}{4}}$$

$$\log_e (1+p) = p - \frac{p^2}{2} + \frac{p^3}{3} + \cdots + \cancel{\frac{(-1)^n p^n}{(n+1)}}$$

$$+ \frac{(-1)^{n-1} p^n}{n}$$

$$\neq$$

$$S_n = a_0 - \frac{a_1}{2} + \frac{a_2}{3} + \cdots +$$

$$I = \int_0^1 \frac{dx}{\sqrt{x(1-x)}}$$

$$x = \sin u$$

$$u = (1-x)$$

$$x = u+1$$

$$dx = du$$

$$= \int_0^1 \frac{dx}{\sqrt{x - x^2}}$$

$$u = x - x^2$$

$$= \int_1^0 \frac{dx}{\sqrt{(u+1)u}}$$

$$I = \int_0^1 \frac{dx}{\sqrt{x(1-x)}}$$

$x^{-1/2}, \quad (1-x)^{-1/2},$

$u = x^{-1/2} \qquad du = -1/_2 x^{-3/2}$

$(1-x)^{-1/2} dx = dv$

$v = \sin^{-1} x$

$$I = x^{-1/2} \sin^{-1} x + \frac{1}{2} \int \sin^{-1} x \; x^{-3/2} \, dx.$$

$du = x^{-3/2} dx \qquad v = \sin^{-1} x$

$u = 2x^{-1/2} \qquad dv = \frac{1}{\sqrt{1-x^2}} dx.$

$du = \sin^{-1} x \, dx$

$u =$

$$u_r = r(r+1)(r+2).$$

$$f(r) = r(r+1)(r+2)(r+3).$$

$$f(r) - f(r-1) = r(r+1)(r+2)(r+3) - (r-1)(r)(r+1)(r+2).$$

$$= r(r+1)(r+2)\{(r+3) - (r-1)\}.$$

$$= r(r+1)(r+2)\{r+3-r+1\}.$$

$$= 4r(r+1)(r+2)$$

$$= 4u_r$$

$$\therefore \quad u_r = \frac{1}{4}\{f(r) - f(r-1)\}.$$

$$S_n = \frac{1}{4}\{f(n) - f(0)\}.$$

$$= \frac{1}{4}\{n(n+1)(n+2) - 0\}$$

$$= \frac{1}{4}n(n+1)(n+2).$$

$u_r = r(r+1)(r+2)$.

$r = 0$

$r = -2$

~~u_r~~

$f(r) = r(r+1)(r+2) + Ar(n+1) + Br + C$.

$r = -1$

$0 = C - B$.

$\therefore C = 0, \therefore B = 0, \therefore A = 0,$

Ex 10a.

1. $1.2 + 2.3 + 3.4 + \ldots + r(r+1)$.

$u_r = r(r+1)$.

$fr = r(r+1)(r+2)$.

$$f(r) - f(r-1) = r(r+1)(r+2) - (r-1)(r)(r+1)$$

$$= r(r+1)\{(\cancel{r}+2) - (\cancel{r}-1)\}$$

$$= r(r+1)\{3\}$$

$$= 3u_r.$$

$S_n = \frac{1}{3}\{f(n) - f(0)\}$.

$\therefore S_n = \frac{1}{3} n(n+1)(n+2)$.

2. $3.4 + 4.5 + 5.6 + \dots + (r+2)(r+3).$

Let $f(r) = (r+2)(r+3)(r+4), \quad u_r = (r+2)(r+3)$

$$f(r) - f(r-1) = \cancel{(r+1)(r+}$$
$$= (r+2)(r+3)(r+4) - (r+1)(r+2)(r+3).$$

$$\cancel{f(r) = (r+2)(r+3) + A(r+2)} + B.$$
$$\cancel{r=0, \; u_r = 0 \; \therefore \; B = 0}.$$
$$\cancel{r=-3, \; u_r = 0 \; \therefore \; A = 0}.$$

$$\cancel{f(r) = (r+2)(r+3)}.$$

$$\cancel{f(r) - f(r-1) = (r+2)(r+3) - f(r+1)(r+2)}$$

$$\cancel{= (r+2)\{r+3-(r+1)\}}$$

$=$

$$f(r) = (r+2)(r+3)(r+4).$$
$$f(r) - f(r-1) = (r+2)(r+3)(r+4) - (r+1)(r+2)(r+3).$$

$$= (r+2)(r+3)\{(r+4) - (r+1)\}.$$

$$= (r+2)(r+3)\{3\}.$$

$$= 3u_r.$$

$$S_n = \tfrac{1}{3}\{ f(n) - f(0)\}.$$

$$= \tfrac{1}{3}\{ (n+2)(n+3)(n+4) - 24 \}.$$

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3. $2.5 + 5.8 + 8.11 + \ldots$

$\cancel{(3n-1)(}$

$$u_r = (3r-1)(3r+2).$$

$$f(r) = (3r-1)(3r+2)(3r+5).$$

$$f(r) - f(r-1) = (3r-1)(3r+2)(3r+5)$$

$3(r-1)$
$3r-3$

$$- (3r-4)(3r-1)(3r+2).$$

$$= (3r-1)(3r+2)\{ 3r+5-3r+4\}$$

$$= 9(3r-1)(3r+2)$$

$$= 9u_r.$$

$$\therefore\ S_n = \tfrac{1}{9}\{ f(n) - f(0)\}.$$

$$= \tfrac{1}{9}\{ (3n-1)(3n+2)(3n+5) + 10\}.$$

$10 - 4 \qquad 8 \downarrow$

4. $1.6 + 6.11 + 11.16 + \ldots$

$\cancel{u_r = (n +}$

$$u_r = (5n-4)(5n+1).$$
$$f(r) = (5r-4)(5r+1)(5r+6).$$

$$f(r) - f(r-1) = (5r-4)(5r+1)(5r+6) - (5r-9)(5r-4)(5r+1)$$

$5r - 5$

$$= (5r-4)(5r+1)\{5r+6-5r+9\}.$$

$$= (5r-4)(5r+1)\{15\}.$$
$$= \cancel{\tfrac{1}{15}/4r}.\ 15u_r.$$

$$u_r = \frac{1}{15}\{f(r)-f(r-1)\}.$$

$$S_n = \frac{1}{15}\{f(n)-f(0)\}.$$

$$= \frac{1}{15}\{(5r-4)(5r+1)(5r+6) + 24\}.$$

5. $\dfrac{1}{1.2} + \dfrac{1}{2.3} + \dfrac{1}{3.4} + \cdots$

$$u_r = \frac{1}{n(n+1)}.$$

$$f(r) = \cancel{[illegible]} \quad \frac{1}{r+1}$$

$$\cancel{f(r) - f(r-1) = \frac{1}{r} - \frac{1}{(r-1)}}.$$

$$f(r) - f(r-1) = \frac{1}{r+1} - \frac{1}{r}$$

$$= \frac{r - (r+1)}{r(r+1)}$$

$$= -\frac{1}{r(r+1)} = -u_r.$$

$$\therefore \quad u_r = -\{f(r) - f(r-1)\}$$

$$\therefore \quad S_n = -\{f(n) - f(0)\}.$$

$$= \cancel{\frac{1}{n(n+1)} +}$$

$$= -\left\{\frac{1}{n+1} - 1\right\}.$$

$$= 1 - \frac{1}{n+1}$$

$$= \frac{n+1-1}{n+1} = \frac{n}{n+1}.$$

$\cancel{2n}$ $(2n+1)$ $\cancel{(2n+1)}$

$(2n+3)$

6. $$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots$$

$$u_r = \frac{1}{(2n+1)(2n+3).}$$

$$f(r) = \frac{1}{(2r+3).}$$

$$f(r) - f(r-1) = \frac{1}{2r+3} - \frac{1}{2r+1}$$

$(2r-2+3)$

$2r+1$

$$= \frac{(2r+1)-(2r+3)}{(2r+1)(2r+3)}$$

$$= \frac{-2}{(2r+1)(2r+3)} = -2u_r.$$

$$\therefore\ u_r = -\frac{1}{2}\left\{f(r) - f(r-1)\right\}.$$

$$\therefore\ S_n = -\frac{1}{2}\left\{\frac{1}{2n+3} - 1/3\right\}.$$

$$= -\frac{1}{2}\left\{\frac{3-(2n+3)}{3(2n+3)}\right\}.$$

$$\therefore\ S_n = \frac{n}{3(2n+3).}$$

7. $1.2.3 + 2.3.4 + 3.4.5 + \ldots$

$u_r = r(r+1)(r+2).$

$f(r) = r(r+1)(r+2)(r+3).$

$f(r) - f(r-1) = r(r+1)(r+2)(r+3) - (r-1)(r)(r+1)($ r

$= r(r+1)(r+2)\{(r+3) - r+1\}.$

$= 4r(r+1)(r+2).$

$= 4u_r.$

$\therefore \quad S_n = \frac{1}{4}\{f(n) - f(0)\}.$

$= \frac{1}{4}\{n(n+1)(n+2)(n+3)\}.$

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8. $1.4.7 + 4.7.10 + 7.10.13$

$u_r = (3n-2)(3n+1)(3n+4)$

$f(r) = (3n-2)(3n+1)(3n+4)(3n+7).$

$f(r) - f(r-1) = (3r-2)(3r+1)(3r+4)(3r+7)$

$3r-3$

$-(3r-5)(3r-2)(3r+1)(3r+4)$

$$= (3r-2)(3r+1)(3r+4)\{(3r+7)-(3r-5)\}.$$

$$= 12(3r-2)(3r+1)(3r+4).$$

$$= 12u_r$$

$r+2)$.

$$\therefore \quad u_r = \frac{1}{12}\{f(r)-f(r-1)\}.$$

$$\therefore \quad S_n = \frac{1}{12}\{(3n-2)(3n+1)(3n+4)(3n+7)+56\}.$$

9. $$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5}$$

$$u_r = \frac{1}{n(n+1)(n+2)}$$

$$f(r) = \frac{1}{(r+1)(r+2)}.$$

$$f(r) - f(r-1) = \frac{1}{(r+1)(r+2)} - \frac{1}{r(r+1)}$$

$$= \frac{r-(r+2)}{r(r+1)(r+2)}$$

$$= -2u_r.$$

$$\therefore \quad u_r = -\frac{1}{2}\{f(r) - f(r-1)\}.$$

$$\therefore \quad S_n = -\frac{1}{2}\{f(n) - f(0)\}.$$

$$= -\frac{1}{2}\left\{\frac{1}{(n+1)(n+2)} - \frac{1}{2}\right\}.$$

$$\cancel{= -\frac{1}{2}\left\{\frac{2-(n+1)(n+2)}{2(n+1)(n+2)}\right\}}.$$

10. $\frac{1}{3.5.7} + \frac{1}{5.7.9} + \frac{1}{7.9.11}$

$$u_r = \frac{1}{(2r+1)(2r+3)(2r+5)}.$$

$$f(r) = \frac{1}{(2r+3)(2r+5)}$$

$$f(r) - f(r-1) = \frac{1}{(2r+3)(2r+5)} - \frac{1}{(2r-1)(2r+3)}$$

$2r-2$

$$= \frac{(2r-1) - (2r+5)}{(2r-1)(2r+3)(2r+5)}$$

$$= \frac{-6}{(2r-1)(2r+3)(2r+5)} = -6u_r$$

$$\therefore \quad u_r = -\frac{1}{6}\left\{ f(r) - f(r-1) \right\}.$$

$$S_n = -\frac{1}{6}\left\{ f(n) - f(0) \right\}.$$

$$= -\frac{1}{6}\left\{ \frac{1}{(2n+3)(2n+5)} - \frac{1}{15} \right\}.$$

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11. $2.3.4.5 + 3.4.5.6 + 4.5.6.7 + \ldots$

$$u_r = (r+1)(r+2)(r+3)(r+4).$$

$$f(r) =$$

$$x^3 + y^3 + z^3 = (x+y+z)^3 = (x^2+y^2+z^2)^3$$

$$\cancel{x^3+y^3+z^3+3xyz} = \cancel{(x+y+z)\tfrac{1}{2}\left[(x-y)^2+(y-z)^2 + (z-x)^2\right]}$$

$$x^3+y^3 = (x+y)(x^2+y^2+xy)$$

$$x^3+y^3+z^3 = (x+y+z)^3 - 3\sum xy - 6xyz.$$

$$3\sum xy + 6xyz = 0$$

$$\cancel{3}(yx+yz+xz+x^2+y^2+z^2) + 6xyz = 0.$$

$$\cancel{3}(yz+yx+xz+2) + \overset{2}{\cancel{6}}xyz = 0$$

$$yz + yx + xz + 2 = -2xyz.$$

$$y(z+x) + xz + 2 = -2xyz.$$

$$\cancel{y(2-y) + xz + 2 = -2xyz}$$

$$\cancel{2y - y^2 + 2 = -2xz}(2y+1)$$

$$\cancel{y^2 - 2y - 2 = xz(2y+1)}$$

$$\cancel{(2-(x+z))(z+x) + xz + 2 = -2xz(2-(x+z))}$$

$$\cancel{2(z+x) - (x+z)^2 + xz + 2 = -4xz + 2xz(x+z)}.$$

$$xz \qquad\qquad -3xz$$

$$\cancel{2z + 2x - x^2 - 2xz - z^2 + xz + 2 = -4xz} + 2x^2z + 2z^2$$

$$\cancel{y^2 - 2y - 2 = x(2-(x+y))(2y}+1).$$

$$\cancel{y^2 - 2y - 2 =}$$

$\frac{p}{q}$ $\quad i$ $\quad a, b$

let nos be

$\left(\frac{a}{c}\right)\left(\left(\frac{b}{a}\right) + \frac{c}{d} q\right)$

a, b, c integers

q irrational

$\frac{p}{q} - i = b$

$i - \frac{p}{q} = K$

i

$3 + 4i$

$ai \quad b + a\sqrt{2}$

11. $2.3.4.5 + 3.4.5.6 + 4.5.6.7 + \dots$

$u_r = (r+1)(r+2)(r+3)(r+4).$

$f(r) = (r+1)(r+2)(r+3)(r+4)(r+5).$

$f(r) - f(r-1) = (r+1)(r+2)(r+3)(r+4)(r+5) - r(r+1)(r+2)(r+3)(r+4)$

$= \cancel{(r+1)(r+}$

$u_r\{r+5-r\} = 5u_r.$

$\therefore\ u_r = \frac{1}{5}\{f(r) - f(r-1)\}.$

$\therefore\ S_n = \frac{1}{5}\{ \cancel{\Sigma} f(n) - f(0)\}.$

224
5
120

$= \frac{1}{5}\{(n+1)(n+2)(n+3)(n+4)(n+5) - 120\}.$

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12. $\frac{1}{1.3.5.7} + \frac{1}{3.5.7.9} + \frac{1}{5.7.9.11} + \dots$

$u_r = \frac{1}{(2r-1)(2r+1)(2r+3)(2r+5)}$

$f(r) = \frac{1}{(2r+1)(2r+3)(2r+5)}.$

$f(r) - f(r-1) = \frac{1}{(2r+1)(2r+3)(2r+5)} - \frac{1}{(2r-1)(2r+1)(2r+3)}$

$2r-2$

$$= \frac{(2r-1) - (2r+5)}{(2r+1)(2r+3)(2r+5)(2r-1)}$$

$$= -6u_r$$

$$\therefore \quad u_r = -\frac{1}{6}\left\{ f(r) - f(r-1) \right\}.$$

$$\therefore \quad S_n = -\frac{1}{6}\left\{ \frac{1}{(2n-1)(2n+1)(2n+3)(2n+5)} + \frac{1}{15} \right\}$$

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14. i). $1.4 + 2.7 + 3.10 + \dots$

$\cancel{\Leftrightarrow}\quad u_r = r(3r+1)$

$\cancel{f(r) = r(3r+1) + Ar + C.}$ $\quad \cancel{r=0,\ C=0.}$

$\cancel{r=-1/3,\ A=0}$

$\cancel{f(r) - f(r-1) = r(3r+1) - (r-1)(3r-2)}.$

$\cancel{3r-3}$

$\cancel{f(r)} = r(3r+1)$

$u_r = \quad r(r+1) + Ar + C.$ $\quad \cancel{-1/3}$

$r=0, \therefore C=0.$

$0 = -1/3\left\{2/3\right\} - \frac{A}{3}$ $\quad r=-1/3$

$= \frac{2-3A}{9}$ $\quad 2-3A = 0 \ \therefore$

$A = 2/3$

$\cancel{f(r) = r(r+1) + 2/3\, r.}$

$\cancel{f(r) - f(r-1) = r(r+1) + 2/3\, r - (r-1)r - 2/3(r-1)}$

$\cancel{= r\{ r+1 + 2/3\, r - r} + 1$

$$\boxed{f(r) - f(r-1) = r(3r+1).}$$

$\cancel{f(r) = r(3r+1)(5r+2).}$

[redacted]

$r(3r+1)(3r+3) - (r-1)(3r$

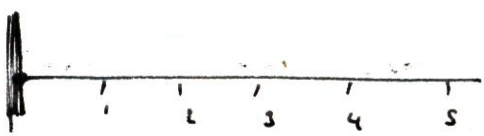
To Prove

any rational number a/b repeats itself.

ie a/b is not irrational but can be infinite + if it is infinite it must repeat ~~that~~ itself.

ie $\cancel{\frac{a}{b}}$ F $\quad \frac{A}{B} = c \leftarrow$ finite

or $\quad \frac{A}{B} = \kappa \leftarrow$ infinite + repeating



$5/4 = 1.25$

$\therefore 5 = 4 \times 1.25.$
$= 4 + 0.25 \times 4$
$= 4 + (4/4)$
$= 4 + 1.$

$\frac{6}{3} = 2 \quad$ ie $\quad 6 = 3 \times 2$
$= 3 + 3.$

$\frac{7}{3} = 2.33333\dot{} \ldots$

$\therefore 7 = 3 \times 2.3333\dot{}$
$= \cancel{3+8}$
$= 3 \times 2 + 3 \times 0.333\dot{}$
$= (3+3) + (3 \times 0.3) + 3 \times (0.03333\dot{}$
$= (3+3) + \left(\frac{3 \times 3}{10}\right) + (3 \times 0.0333)$

$$7 = (3+3) + \frac{3\times 3}{10} + 3\times 0.\dot{3}33\cdots$$

$$= (3+3) + \frac{3+3+3}{10} + 3\times 0.0333\cdots$$

$$= (3+3) + \frac{3+3+3}{10} + \frac{3\times 0.03}{} + 3\times 0.00333\cdots$$

$$7 = 3+3 + \frac{3+3+3}{10} + \frac{3+3+3}{100} + \frac{3+3+3}{1000} + \cdots$$

$$\frac{12}{7} = 1.714\ 2857\ldots$$

$$\therefore\ 12 = 7\times\{1.7142857\ldots\}.$$

$$= 7\times 1 + \{7\times 0.142857\ldots\}.$$

$$= \cancel{7\times}$$

$$= 7 + \frac{7}{10} + 7\times 0.042857\ldots$$

$$= 7 + \frac{7}{10} + \frac{7\times 4}{100} + \frac{7\times 2}{1000} + \frac{7\times 8}{10000}\ldots$$

$$12 = 7 + \frac{7}{10} + \frac{7+7+7+7}{100} + \frac{7+7}{1000}\ \ldots$$

$$\frac{A}{B} = C + \dot{J}$$

$\dot{J}$ = an infinite ~~no~~ +ve real no. < 1

if repeats ~~itself~~ ~~that~~ itself.

$$\frac{A}{B} - C = \dot{J}$$

$$\frac{\dot{J}}{10^{n}} = \dot{J} - \frac{\dot{J}}{10^{n}}$$ if it repeats after n

$$\therefore \dot{J} = \frac{2\dot{J}}{10^{n}}$$

$0.1234\,1234\,1234 \ldots = \dot{J}$

$\frac{\dot{J}}{10^{4}} = 0.0000\,1234\,1234\,1234 \ldots$

$$\dot{J} - \frac{\dot{J}}{10^{4}} = \begin{array}{r} 0.1234\,1234\,1234 \\ 0.0000\,1234\,1234 \\ \hline 0.1234 \end{array}$$

r is the no. that is reapeated.

$$\therefore \dot{J} = r + \frac{\dot{J}}{10^{n}}$$

$$\therefore \frac{A}{B} = r + \frac{\dot{J}}{10^{n}}$$

~~th~~ true for any rational no.

$$\left\{ \frac{A}{B} = r + \frac{j}{10^n} \right. \qquad \begin{array}{l} r \to \text{finite} \\ j \to \text{infinite} \\ n \to \text{const.} \end{array}$$

$$\frac{A}{B} = \sqrt{2} \qquad \frac{A^2}{B^2} = 2$$

$$\frac{A}{B} = r + \frac{j}{10^n} \qquad \frac{A^2}{B^2} = \left\{ r^2 + \cancel{j}\, \frac{2r^2j^2}{10^{2n}} + \frac{j^2}{10^{2n}} \right\}.$$

$$2 = r^2 + \frac{2r^2j^2}{10^{2n}} + \frac{j^2}{10^{2n}}$$

$$\frac{A}{B} = r + \frac{j}{10^n} \leftarrow \text{rational}$$

rational

~~fixed~~ ~~value~~ (a number)

if $\frac{A}{B}$ is finite then j is finite.

$\therefore$ $\frac{j}{10^n}$ is finite $\therefore$ r is finite.

if $\frac{A}{B}$ is infinite j is infinite

$\frac{j}{10^n}$ is infinite

and r is finite or infinite

but $\frac{A}{B}$ is rational $\therefore$ it must be finite

$$\cancel{r = \frac{A}{B} - \frac{j}{10^n}}$$

$$\cancel{= j - \frac{j}{10^n} = \frac{10^n j - j}{10^n}}$$

$$\cancel{r = \frac{j(\cancel{1 - 10^n})\,10^n - 1}{10^n}}$$

$$\frac{A}{B} - c = j = r + \frac{j}{10^n}$$

$$\cancel{\therefore r = \frac{A}{B} - c - \frac{j}{10^n}}$$

$$\cancel{= j - \frac{j}{10^n} - c}$$

$$\cancel{= \frac{j(10^n - 1)}{10^n} - c}$$

$$r = \frac{A}{B} - c - \frac{j}{10^n} = j - \frac{j}{10^n}$$

$$= \frac{j\,(10^n - 1)}{10^n}.$$

if $\frac{A}{B}$ is rational and j is infinite then r must be finite.

$$j = \frac{A}{B} - C$$

$$r = \frac{\left\{\frac{A}{B} - C\right\}\left\{10^n - 1\right\}}{10^n}$$

$$= \frac{(A - BC)\left\{10^n - 1\right\}}{B \cdot 10^n}$$

$$\cancel{= \frac{10^n A - A - BC\,10^n - BC}{B\,10^n}}$$

$$r = \frac{(A - BC)(10^n - 1)}{B \cdot 10^n}$$

A, B, C, integers such that $\frac{A}{B}$

5, 11, 17

if $\frac{A - BC}{B}$ is ~~an~~ an infinite no. i

$$A - BC = Bi$$

A is finite, B is finite
C is ~~fin~~ finite.

$$A = f(x)$$
$$B = g(x)$$

$$\frac{A}{B} = \frac{f(x)}{g(x)}$$

$$f(x) = (x-a)\,g(x) + R.$$

$$\frac{A}{B} = R + i$$

$$\frac{f(x)}{g(x)} = (x-a) + \frac{R}{g(x)}$$

$$R = abcde\ldots\ldots abcde\ldots$$
$$= a + b\times 10^{-1} + c\times 10^{-2} + d\times 10^{-3} + e\times 10^{-4} + \ldots + a\times 10^{-n} + b\times 10^{-(n+1)} + c\times 10^{-(n+2)}$$
$$= a(1+10^{-n}) + b(10^{-1} + 10^{-(n+1)}) + c(10^{-2} + 10^{-(n+2)}) + \ldots$$
$$= a(1+10^{-n}) + b\times 10^{-1}(1+10^{-n}) + c\times 10^{-2}(1+10^{-n}) + \ldots$$
$$= (1+10^{-n})(a + b\times 10^{-1} + c\times 10^{-2} + \ldots$$

$\frac{A}{B}$ is finite or infinite

if $\frac{A}{B}$ is infinite then it must repeat after a certain no. of digits.

$$\frac{A}{B} = c + i + r + r \times 10^{-n} + r \times 10^{-2n} + \ldots$$

rational no > 0

~~irrational~~ finite no $1 > i \geq 0$

finite no $1 > r \geq 0$ and $i > r$

$$\frac{A}{B} + i = \frac{A + iB}{B}$$

$$\int x^2 e^{x^3}\,dx \qquad u = x^3 \qquad du = 3x^2\,dx$$

$$I = \frac{1}{3}\int e^u\,du$$

$$t = \frac{1}{1-4\sqrt{2}}$$

$$= \frac{(1+4\sqrt{2})}{(1-4\sqrt{2})(1+4\sqrt{2})} = \frac{(1+4\sqrt{2})}{(1-\sqrt{2})}$$

$$= \frac{(1+4\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = \frac{(1+4\sqrt{2})(1+\sqrt{2})}{1-2}$$

$$= (1-4\sqrt{2})(1+\sqrt{2}).$$

$$1 + 1 + 1 + 1 = 4$$
$$= 0$$
$$= 0$$
$$= -4$$

$$(3x-1)^7 = a_7 x^7 + a_6 x^6 + \cdots + a_0$$

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}
$$

$(3.5)\ (2.5)$

$$(3x-1)^7 = -1 + 3.7 - 3^2.21 + 3^3.35 - 3^4.35 + 3^5.21 - 3^6.7 + 3^7$$

$$= 3.7\{1 - 3^5\} + 3^2.21\{-1 + 3^3\}$$

$$(x+a)(y+b) = xy + ay + bx + ab$$

$A = 16 + B + C$

$A^2 = 1632 + B^2 + C^2$

$\boxed{A + B + C}$

$A^2 = (16 + B + C)^2$

$= 16^2 + B^2 + C^2 + 2\Sigma \cdot AB$

$= 16^2 + B^2 + C^2 + 2\{16B + 16C + CB\}$

$A^2 = 16^2 + B^2 + C^2 + 32B + 32C + 2CB.$

$\cancel{16 = A - B - C}$

$\cancel{16^2 = A^2 + B^2 + C^2 + 2\Sigma AB}$

$$\begin{array}{r} 16 \\ \times 16 \\ \hline 96 \\ 160 \\ \hline 256 \end{array}$$

$k = \cos 45 + i \cos 135 + i^2 \cos 215 + \ldots$

$=$

$\log_2 3$

$\frac{1}{6}$

$\frac{1}{6}$ or $2/6$ 3.4.

3.2

$\frac{2}{6}$

$\frac{2}{3}$ × $2/6$ ×

2.3

1 2 3

6 5 4

$$y = \cancel{(\log_2 1)}(\log_3 4)\ldots$$

$$= (\log_3 2)(\log_3 4)\ldots$$

$$= \frac{\log_3 4}{\log_3 2} = 2.$$

$$y = (\log_2 3)(\log_3 4)(\log_4 5)(\log_5 6)\ldots$$

$$= \frac{\log_3 4}{\log_3 2} \cdot \frac{\overset{\log_5 6}{\cancel{\log_4 5}}}{\log_5 4} \cdot \frac{\log_7 8}{\log_7 6} \ldots$$

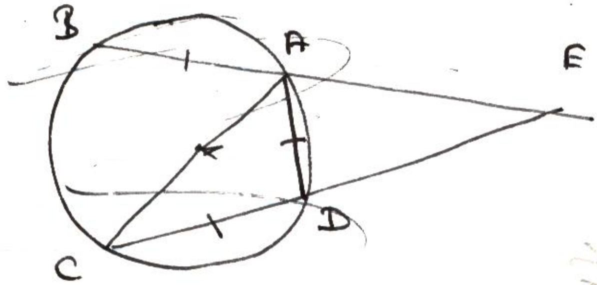
$$\frac{6}{4} \qquad \frac{8}{6}$$

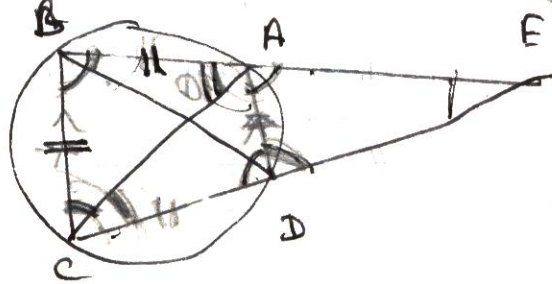
$$\cancel{= 2 \cdot \frac{\log_4(3\times 2)}{\log_5 4}} \cdot \frac{\log_7(4\times 2)}{\log_7(3\times 2)}$$

$$y = (\log_2 3)(\log_3 4)\ldots(\log_n(n+1))\ldots\log_{31} 32.$$

$$(a+1)x + (a+1) = 0$$

(-1)





$x^2 + x + 1 = 0$

$x^2 - x - 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

2.

$(x-\alpha)(x-\beta)=0$

$(x-\alpha)(x-\gamma)=0$

$\alpha+\beta=-a \quad , \quad \alpha\beta=1$

$\alpha+\gamma=1 \quad \alpha\gamma=-1$

$\beta=-(\cancel{\gamma-a})-a-\alpha$

$\gamma=1-\alpha$

$\beta+\gamma=-a+1=1-a$

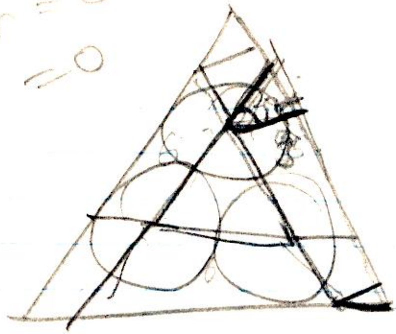
$x^2+ax+1=0 \quad x=\dfrac{-a\pm\sqrt{a^2-4}}{2}$

$x^2-x-a=0 \quad x=\dfrac{1\pm\sqrt{1+4a}}{2}$

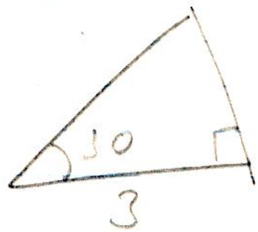
$-a\pm\sqrt{a^2-4}=1\pm\sqrt{1+4a}$

$x^2 + mx + n = 0$

$x^2 + px + q = 0$



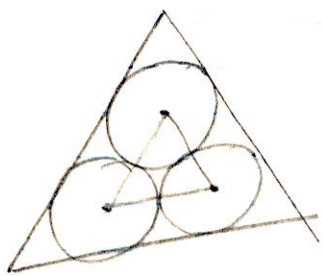
18 +



$\cos 30 = \frac{\sqrt{3}}{2}$

$3 \cdot \sqrt{3}$

$3 \frac{\sqrt{3}}{2}$



$18 + 18 \frac{\sqrt{3}}{2}$

$x^2 + px + q = 0 \leftarrow \alpha^3, \beta^3$

$x^2 + mx + n = 0 \quad \alpha, \beta$

$\alpha + \beta = -m \qquad \alpha^3 + \beta^3 = -p$

$\alpha\beta = n \qquad \alpha^3\beta^3 = -q$

$\cancel{\alpha^3 + \beta^3} =$

$(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\Sigma\alpha\beta$

$-p = (-m)^3 - 3(n)$

$-p = -m^3 - 3n$

$p = m^3 + 3n$

$(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$-p = (-m)^3 + 3nm$

$p = m^3 - 3mn$

$g(x) = x^5 + x^4 + x^3 + x^2 + x + 1$

$(x-1)g(x) = x^6 - 1$

$$\frac{(x-1)g(x^{12})}{(x-1)g(x)} = \frac{(x^{12})^6 - 1}{x^6 - 1}$$

$$\frac{g(x^{12})}{g(x)} = \frac{(x^{12})^6 - 1}{x^6 - 1}$$

$$= \frac{(x^{12 \cdot 3} - 1)(x^{12 \cdot 3} + 1)}{(x^3 - 1)(x^3 + 1)}$$

$$= \frac{(x^{6 \cdot 3} - 1)(x^{6 \cdot 3} + 1)(x^{12 \cdot 3} + 1)}{(x^3 - 1)(x^3 + 1)}$$

$4 - x^2 + 1$

$3 - x + 1 = \cancel{x} + 1$

$2 - \cancel{x} + 1$

x^{12}

x^3

$\frac{6}{5}$ 1

$$\frac{x^2 + 2}{x - 1} = x + 1 - \frac{3}{x - 1}$$

$$\frac{6}{1} = 6 \quad \frac{3}{4}$$

$(x - 3)^2 =$

$$u_r = \frac{1}{(2n-1)(2n+1)}$$

$$f(n) = \frac{1}{(2n+1)}$$

$$f(n) - f(n-1) = \cancel{\frac{1}{(2n+1)}} - \frac{1}{}$$

$2n-2$

$2n-1$

$$= \frac{1}{(2n-1)} - \frac{1}{(2n+1)}$$

$2n-2$

$$= \frac{2n+1-2n+1}{(2n-1)(2n+1)}$$

$$= 2u_r$$

$$u_r = \frac{1}{2}\left\{f(n) - f(n-1)\right\}$$

$$S_n = \frac{1}{2}\left\{f(n) - f(0)\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{255.277} - \frac{1}{3}\right\}$$

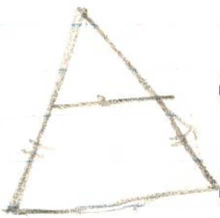
$$= \frac{3 -}{3 \times 255 \times 277}$$

$b = 2a \cos 20$

$d = 2b \cos 40$

$d = 4a \cos 20 \cos 40$

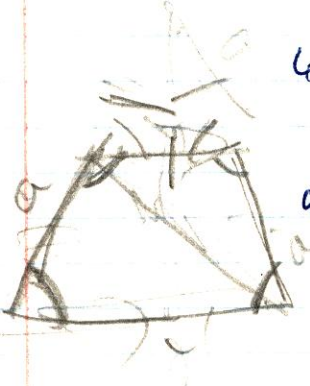
$= 2a \{\cos 60 + \cos 30\}$



$4a \cos 20 \cos(2 \times 20)$

$4a \{\cos 20 [\cos^2 20 - \sin^2 20]\}$

$4a \{\cos^3 20 - \sin^2 20 \cdot \cos 20\}$



$4a \{\cos^3 20 -$

$c < a+b$

$d < c+a$

$d - 4a \leq$

$d < 2a+b$

$2b = a+d$

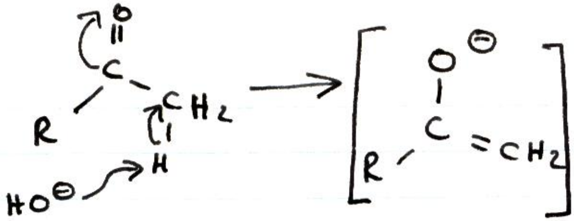
~~A~~ B C D E

Finally

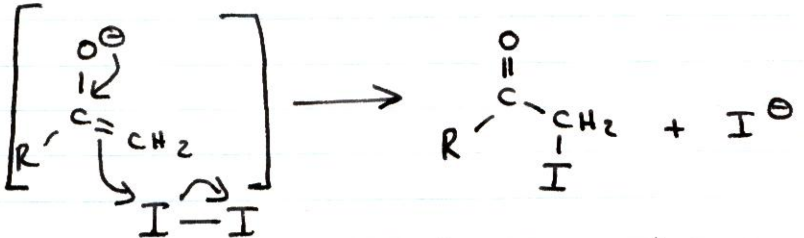
$$^{\ominus}CI_3 + H{-}OH \rightarrow CHI_3 + OH^{\ominus}.$$

and

$$R{-}\overset{\displaystyle O}{\overset{\|}{C}}{-}O{-}H + {}^{\ominus}OH \rightarrow R{-}\overset{\displaystyle O}{\overset{\|}{C}}{-}O^{\ominus} + H_2O.$$



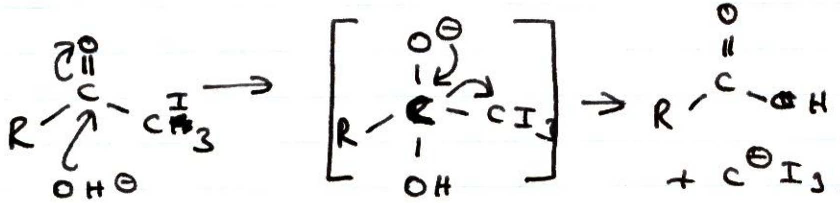
an enolate anion.
(quite a likely ion in an alkaline soln.).



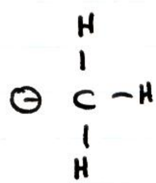
In 2 more similar steps we get :-

$$RCOCH_2I \longrightarrow\longrightarrow RCOCI_3.$$

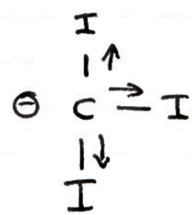
The next step involves attack by an $OH^{\ominus}$ on the carbonyl C



bad



charge cannot be delocalised



good because charge can be delocalised due to the inductive effect of the iodines.

an alternative reagent is NaOCl / KI(aq)
ie ~~sta~~ sodium chlorate(I) solution (made by bubbling Cl_2 into cold dil. NaOH (aq)) + KI soln. The solution effectively contains $OI^{\ominus}$(aq) an oxidising agent and a source of I_2.

The reaction is :-

$$\boxed{R-COCH_3 + 3I_2 + 4NaOH \rightarrow CHI_3\downarrow + 3NaI + RCO_2Na + 3H_2O.}$$

or

$$RCOCH_3 + 3I_2 + 4OH^- \rightarrow CHI_3 + 3I^{\ominus} + RCO_2^{\ominus} + 3H_2O$$

WITH NaOCl / KI equn. is :-

$$RCOCH_3 + 3OI^{\ominus} \rightarrow CHI_3 + RCO_2^{\ominus} + 2OH^{\ominus}$$

$$[OCl^- + I^{\ominus} \rightarrow OI^{\ominus} + Cl^{\ominus}].$$

Mechanism

The mechanism of the ~~trior~~ triiodomethane reaction involves a progressive substitution of I for H on the terminal CH_3 group.

Kinetic Theory of Liquids The molecules in a liquid are in a state of random motion as in the gas the motion is attenuated and the molecules in a liquid much closer together. Liquids lye mid way between the disorderly scattered arrangement in gas + the orderly compact arrangement found in solids Because of the closeness the attractive forces are stronger Liquid molecules have restricted translational motion + no of col / sec in liquid is much greater than in a gas ($\approx 1000 \times 10^6$ / sec.) random motion of molecules in a liquid — Brownian movement ie the random motion of small pollen grains in water due to molecular bombardment.

Joule - Kelvin Effect.

For most gases it is observed that if a gas is allowed to expand through a porous plug it becomes cooled at ordinary temps. H_2 becomes heated and the difference is due to the shape of the PV

curves

Most gases resemble N in that the product $PV >$ at lower pressure for normal p ranges. PV has units of energy and because PV is greater @ lower pressures most gas on expanding through a plug have to do work. There are 2 reasons

PV

1). The gas has to push away other gas molecules away infront of it.

2). To overcome intermolecular attraction forces.

PV for H_2 from high to low gets smaller. Since the expansion is rapid energy cannot be absorbed from the surroundings so the gas molecules lose $K.E \propto T$. Provided it is brought below a certain T before being allowed to expand through a plug H also shows a cooling effect + this temp is called the inversion temp.

$\rho = N_m$