CITY OF LIVERPOOL

EDUCATION COMMITTEE

NAME J. ALLDAY

FORM or CLASS L6 Sc(1)

SUBJECT F. MATHS (M. CHAMPION)

Ex 3b.

$$\cancel{1.\ \begin{vmatrix} 17 & 20 \\ 15 & 19 \end{vmatrix} = 5^2 \begin{vmatrix} 17 & 4 \\ 3 & 19 \end{vmatrix}}$$

$$\cancel{= 25.(17.19 - 12)}$$

$$\cancel{= 7775.}$$

1. $\begin{vmatrix} 17 & 20 \\ 15 & 19 \end{vmatrix} = 23.$

2. $\begin{vmatrix} 57 & 55 \\ 38 & 44 \end{vmatrix} = 11 \begin{vmatrix} 57 & 5 \\ 38 & 4 \end{vmatrix}$

$$= 11 \times 38 = 418.$$

3. $\begin{vmatrix} 102 & 102 \\ 76 & 78 \end{vmatrix} = 102 \begin{vmatrix} 1 & 1 \\ 76 & 78 \end{vmatrix} = 102.2$

$$= 204.$$

4. $\begin{vmatrix} 201 & 132 \\ 100 & 67 \end{vmatrix} = 3 \begin{vmatrix} 67 & 44 \\ 100 & 67 \end{vmatrix}$

$$= 3.(67^2 - 440)$$

$$= 12147.$$

5. $\begin{vmatrix} 10 & 20 & 30 \\ 5 & 50 & -1 \\ 0 & 60 & 4 \end{vmatrix} = 10^2 \begin{vmatrix} 1 & 2 & 3 \\ 5 & 5 & -1 \\ 0 & 6 & 4 \end{vmatrix}$

$$= \cancel{10^2} \{$$

$$= 10^2 \{ 20 + 90 \cancel{+} - 40 + 6 \}.$$

1 2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~

11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17

~~18~~ 19 ~~20~~

1 2 3 → 6

9

27

54 81

=

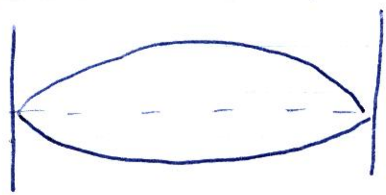
QUANTUM MECHANICS

when we make a model of an atom we must take into account the fact that moving electrons do not behave classically. To help us understand the model we take a model that is simpler: a vibrating string in one ~~dbration~~. direction.

allowed to freely vibrate in the middle

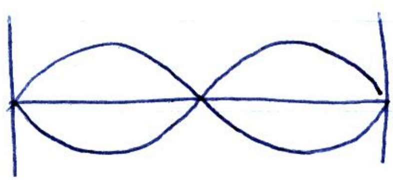
Is possible to set up standing waves

$\lambda = 2l/\cancel{4}$



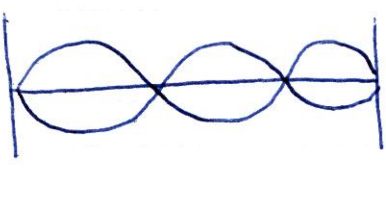
Fundamental.

$\lambda = l$



1st harmonic.

$\lambda = \frac{2l}{3}$



2nd harmonic etc.

λ

Only certain standing waves ie those that last for any length of time, others die because of interference The system is quantised - only certain vibrations will work, as you get to the higher harmonics the energy goes up $\therefore$ only certain energy values are allowed ie not continuous.

Displacement of string at any point (A/2)
$= \phi(x,t)$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} \qquad v = \text{velocity}$$

$$\phi(x,t) = \cancel{A \sin(vt} +$$
$$= \psi(x) \times \sin 2\pi \nu t$$

$\nu \downarrow$ frequency.

$$\phi(x,t) = 2A \sin\left(\frac{n\pi x}{l}\right) \sin 2\pi \nu t$$

$$\psi(x) = 2A \sin \frac{n\pi x}{l}$$

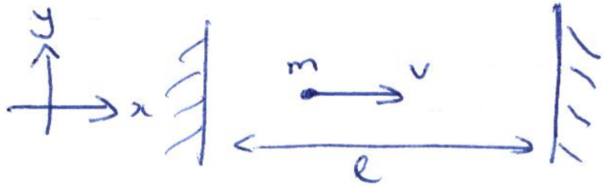
$$\phi(x,t) = \psi(x) \times F(t).$$

$\psi(x)$ is an amplitude function with respect to x. It is a wave function (if t is kept constant). $\psi^2(x)$ ~~.....~~ is proportional to the intensity of the wave at any given x

∂ δ ψ

value.

Consider an electron moving between 2 walls in diamention



It is possible to consider this system like a vibrating string. End up with similar equations: it has to have only certain energy value: it is quantised

$\psi(x)$ ~~A~~ $\psi^2(x)$ – tells you the probability of finding of the electron at a given point x.

Wave mechanics of 3d atoms.

In 1926 Schrödinger and Heisenberg independently laid the foundations for a new sort of mechanics which expressed the wave/particle duality of matter,

Schrödingers model for Hydrogen

Considered hydrogen to be an electron moving around a stationary proton. Between the 2 there is a force of attraction $\left(-\frac{e^2}{r}\right)$

that is the potential energy of the system. K.E

$= \frac{p^2}{2m}$ p = momentum.

E of system $= \frac{p^2}{2m} - \frac{e^2}{2r}$

wanted to find out what amplitude of electron wave in 3 dimentions

$\psi(x, y, z) \Rightarrow$ wave function.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E - U)\psi = 0$$

E = total energy U = potential energy

9. $\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 10 & 15 & 20 \end{vmatrix} = 5 \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 \end{vmatrix}$

~~R1 - R4~~ $\qquad$ C4 - C1

~~R2 - R3~~ ~~R3 - R2~~

R3 - R4

$$= 5 \begin{vmatrix} 2 & 3 & 4 & 3 \\ 3 & 4 & 5 & 3 \\ 4 & 5 & 6 & 3 \\ 1 & 2 & 3 & 3 \end{vmatrix}$$

C3 - C2

$$= 15 \begin{vmatrix} 2 & 3 & 4 & 1 \\ 3 & 4 & 5 & 1 \\ 4 & 5 & 6 & 1 \\ 1 & 2 & 3 & 1 \end{vmatrix}$$

$$= 15 \begin{vmatrix} 2 & 3 & 1 & 1 \\ 3 & 4 & 1 & 1 \\ 4 & 5 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{vmatrix} = \underline{\underline{0}}$$

10. $2 \begin{vmatrix} 2 & 4 & 6 & 8 \\ 0 & 3 & 6 & 9 \\ 9 & 6 & 3 & 0 \\ -8 & -6 & -4 & -2 \end{vmatrix} = 4 \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 9 \\ 9 & 6 & 3 & 0 \\ -4 & -3 & -2 & -1 \end{vmatrix}$

$$= 36 \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \\ -4 & -3 & -2 & -1 \end{vmatrix}$$

$$= \underline{0}$$

$R_1 - R_2 \quad \cancel{R_2} \quad \cancel{R_3 - R_1}$

11. $\begin{vmatrix} 5 & 4 & 2 & -1 \\ 6 & 5 & 2 & 1 \\ 7 & 6 & 2 & -1 \\ 8 & 7 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -1 & -1 & 0 & -2 \\ 6 & 5 & 2 & 1 \\ 7 & 6 & 2 & -2 \\ 1 & 1 & 0 & 2 \end{vmatrix}$ $R_4 - R_3$

$= 0.$

12. $2\begin{vmatrix} a+b & a^2+b^2 \\ 2(a+b) & 2(a^2+b^2) \end{vmatrix} = 0.$

13. $\begin{vmatrix} x-2y & x+2y \\ x+y & x-y \end{vmatrix}$

$= \cancel{\begin{vmatrix} -x & x \\ x+y & x-y \end{vmatrix}} + \cancel{\begin{vmatrix} 2y & 2y \\ x+y & x-y \end{vmatrix}}$

$= \cancel{-x(x-y) - x(x+y)}$

$=$

$(x-y)(x-2y) - (x+y)(x+2y)$

$= x^2 - 2xy - yx + 2y^2 - (x^2 + 2yx + xy + 4y^2$

$= -6xy.$

14. $\begin{vmatrix} a^2-b^2 & (a-b)^2 \\ a(a+b) & b(a-b) \end{vmatrix}$

$= \begin{vmatrix} (a+b)(a-b) & (a-b)(a-b) \\ a(a+b) & b(a-b) \end{vmatrix}$

$= (a+b)(a-b) \begin{vmatrix} (a-b) & (a-b) \\ a & b \end{vmatrix}$

$= a^2-b^2 \left(b(a-b) - a(a-b) \right)$

$= (a-b)(b-a)(a^2-b^2)$

$= (a-b)^2 (a+b)(b-a)$

$= (a-b)^2 (b^2-a^2).$

$(a-b)^2$

$(a-2b)^2$

15. $\begin{vmatrix} a^2-4b^2 & a^2-ab-2b^2 \\ a^2-b^2 & a^2-2ab+b^2 \end{vmatrix}$ $(a-2b)(a+b)$

$= \begin{vmatrix} (a-2b)^2 & (a-2b)(a+b) \\ (a-b)(a+b) & (a-b)^2 \end{vmatrix}$

$= (a-2b)(a-b) \begin{vmatrix} a-2b & a+b \\ a+b & a-b \end{vmatrix}$

$= (a-2b)(a-b) \left\{ (a-b)(a-2b) - (a+b)^2 \right\}.$

$= (a-2b)^2(a-b)^2 - (a-2b)(a^2-b^2)(a+b)$

16. $$\begin{vmatrix} a^2 & a & 1 \\ a & 1 & a^2 \\ 1 & a^2 & a \end{vmatrix} = \cancel{(a-1)(a-1)(a-1)}$$

17. $$2\begin{vmatrix} 2x & x^2 & x^3 \\ 2y & y^2 & y^3 \\ 2z & z^2 & z^3 \end{vmatrix}$$

$$= 2xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= 2xyz\,(x-y)\,(y-z)\,(z-x).$$

18. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ (1+a) & (1+b) & (1+c) \end{vmatrix} = \text{[illegible]}.$$

$(a = b = c = 1)$

19. $$\begin{vmatrix} 2-x & 2 & 3 \\ 2 & 5-x & 6 \\ 3 & 4 & 10-x \end{vmatrix}$$

20. $\begin{vmatrix} a & 0 & a & a \\ a & a & 0 & a \\ a & a & a & 0 \\ 0 & a & a & a \end{vmatrix} = a^4 \begin{vmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix}$

$= a^4 \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \end{vmatrix}$

$= a^4 \begin{vmatrix} 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix}$ $R_1 - R_3$

$= a^4 \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$ $C_1 - C_3$

$= a^4 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{vmatrix}$

$= a^4 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$

$= 2a^4$

$R_2 - R_3$
$R_3 - R_1$

$= a^4 \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{vmatrix}$

$= a^4 \begin{vmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$= a^4 (1 + 1 + 1) = \underline{3a^4}$

21. $$\begin{vmatrix} a^3 & a^2 & a & 1 \\ a^2 & a & 1 & a^3 \\ a & 1 & a^3 & a^2 \\ 1 & a^3 & a^2 & a \end{vmatrix} = \cancel{(a-1)}$$

$(a=0)$

$\{\alpha \neq 8\}$ $\cancel{(a-1)}$

$\cancel{(a-1)\,(a^2-1)\,(a^3}-1)$

22. $$\begin{vmatrix} x+1 & x-1 & x & 1 \\ 2x+2 & x-2 & x^2 & 2 \\ 3x+3 & x-3 & x^3 & 3 \\ 4x+4 & x-4 & x^4 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} x & x-1 & x & 1 \\ 2x & x-2 & x^2 & 2 \\ 3x & x-3 & x^3 & 3 \\ 4x & x-4 & x^4 & 4 \end{vmatrix} + \begin{vmatrix} 1 & x-1 & x & 1 \\ 2 & x-2 & x^2 & 2 \\ 3 & x-3 & x^3 & 3 \\ 4 & x-4 & x^4 & 4 \end{vmatrix}$$

$$+ \begin{vmatrix} 1 & x & x^2 & 1 \\ 2 & x & x^2 & 2 \\ 3 & x & x^3 & 3 \\ 4 & x & x^4 & 4 \end{vmatrix} + \begin{vmatrix} 1 & -1 & x & 1 \\ 2 & -2 & x^2 & 2 \\ 3 & -3 & x^3 & 3 \\ 4 & -4 & x^4 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} x & x & x^2 & 1 \\ 2x & x & x^2 & 2 \\ 3x & x & x^3 & 3 \\ 4x & x & x^4 & 4 \end{vmatrix}$$

$$= \underline{0}$$

$$a^3+b^3 = (a+b)(a^2 - ab + b^2)$$

23. $$\begin{vmatrix} a^3+b^3 & a(a^2-ab+b^2) \\ b^3-a^3 & b(a^2+ab+b^2) \end{vmatrix}$$ $b^3-a^3 = (b-a)(b^2+ab+a^2)$

$$= \cancel{(a+b)^3 - 3ab(a+b)}$$

$$\cancel{= \begin{vmatrix} a^3+b^3 & a^3 - a^2b + ab^2 \\ b^3-a^3 & a^2b + ab^2 + b^3 \end{vmatrix}}$$

$$\begin{vmatrix} a^3+b^3 & a(a^2-ab+b^2) \\ b^3-a^3 & b(a^2+ab+b^2) \end{vmatrix}$$

$$= \begin{vmatrix} (a+b)(a^2-ab+b^2) & a(a^2-ab+b^2) \\ (b-a)(b^2+ab+a^2) & b(a^2+ab+b^2) \end{vmatrix}$$

$$= (a^2-ab+b^2)(a^2+ab+b^2)\begin{vmatrix} a+b & a \\ b-a & b \end{vmatrix}$$

$$= a^4 + \cancel{a^3b} + \cancel{a^2b^2} - \cancel{a^3b} - \cancel{a^2b^2} - \cancel{ab^3} + a^2b^2 + \cancel{ab^3} + b^4$$

$$= (a^4+a^2b^2+b^4)(b(a+b) - a(b-a))$$

$$= (a^4+a^2b^2+b^4)(\cancel{b(a+b)} - $$

$$(ab+b^2-ab+a^2)$$

$$= (a^4+a^2b^2+b^4)(b^2+a^2).$$

24. $$\begin{vmatrix} a+b+c & a+b-c & a-b+c \\ b & -c & a \\ c & a & -b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b-c \\ b-c+a & \\ c+a-b & \end{vmatrix}$$

$$C_2 - C_3 \begin{vmatrix} a+b+c \\ b \\ c \end{vmatrix}$$

$$\begin{array}{ccc} (b+c) & (a-c) & (a-b) \end{array}$$

$$\begin{vmatrix} a+(b+c) & (a+(b-c)) & (a-b+c) \\ c+b & a-c & a-b \\ c & a & -b \end{vmatrix}$$

$$\begin{vmatrix} a+b+c & 2(a+b) & 2(a+c) \\ b & b-c & b+a \\ c & a+c & c-b \end{vmatrix}$$

$$\begin{vmatrix} a+b+c \\ b \\ c \end{vmatrix}$$

$$\begin{vmatrix} a+b+c & a+b-c & a-b+c \\ b & -c & a \\ c & a & -b \end{vmatrix}$$

$$\begin{vmatrix} a+b+c & a+b-c & a-b+c \\ b & -c & a \\ c & a & -b \end{vmatrix} \quad R_2 + R_3$$

$$\cancel{-\begin{vmatrix} a+b+c & a+b-c & a+b+c \\ b+c & a-c & a+b \\ c & a & b \end{vmatrix}}$$

$$\cancel{\neq \begin{vmatrix} a & b & c \\ b & -c & a \\ c & a & -b \end{vmatrix} + \begin{vmatrix} b+c & a-c & a-b \\ b & -c & a \\ c & a & b \end{vmatrix}}$$

$$= \begin{vmatrix} a & b & c \\ b & -c & a \\ c & a & -b \end{vmatrix} + \cancel{\begin{vmatrix} b+c & a-c & a-b \\ -b & -c & a \\ c & a & -b \end{vmatrix}}$$

$$= \begin{vmatrix} a & b & c \\ b & -c & a \\ c & a & -b \end{vmatrix} + \begin{vmatrix} b & -c & -b \\ b & -c & a \\ c & a & -b \end{vmatrix} + \begin{vmatrix} c & a & a \\ b & -c & a \\ c & a & b \end{vmatrix}$$

25. $\begin{vmatrix} x & 2 & 3 \\ -4 & -2x & 2 \\ 2 & x & 1 \end{vmatrix} = 0$

$\therefore \; 2 \begin{vmatrix} x & 2 & 3 \\ -2 & -x & 1 \\ 2 & x & 1 \end{vmatrix} = 0$

$\begin{vmatrix} -x & 2 & 3 \\ -2 & -x & 1 \\ 2 & x & 1 \end{vmatrix} = 0$

$x = \pm 2,$

26. i). $\begin{vmatrix} a & a & a & a \\ b & b & b & -b \\ c & c & -c & -c \\ d & -d & -d & -d \end{vmatrix} = 8abcd$

$\begin{vmatrix} a & a & a & a \\ b & b & b & -b \\ c & c & -c & -c \\ d & -d & -d & -d \end{vmatrix} = abcd \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{vmatrix}$

$R_1 + R_4 \qquad = abcd \begin{vmatrix} 2 & 0 & 0 & 0 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{vmatrix}$

$= 2abcd \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{vmatrix}$ $\quad \cancel{C_3 + C_1}$

$C_2 + C_3$

$C_3 + C_1$

$$= 2abcd \begin{vmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & -2 & -2 \end{vmatrix}$$

$$= 2abcd \begin{vmatrix} -2 & 0 \\ -2 & -2 \end{vmatrix}$$

$$= \underline{8abcd}$$

ii). $$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x-1)^3$$

27. i) $$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \quad \begin{matrix} C_1 + C_2 \\ C_2 + C_3 \end{matrix}$$

$$\begin{vmatrix} b+c & a+b+c & 2a \\ b & a+b+c & a+b+c \\ c & 2c & a+b+c \end{vmatrix}$$

$$\begin{vmatrix} 2a+b+c & a+b+c & 2a \\ a+2b+c & a+b+c & a+b+c \\ 2c+a+b & 2c & a+b+c \end{vmatrix}$$

$$= \begin{vmatrix} a & 0 & a \\ b & 0 & 0 \\ c & c & 0 \end{vmatrix} + \begin{vmatrix} a+b+c & a+b+c & a \\ a+b+c & a+b+c & a+b+c \\ a+b+c & c & a+b+c \end{vmatrix}$$

$$= -c \begin{vmatrix} a & a \\ b & 0 \end{vmatrix} + (a+b+c)^2 \begin{vmatrix} 1 & a+b+c & a \\ 1 & 1 & 1 \\ 1 & c & a+b+c \end{vmatrix}$$

$$= -c \begin{vmatrix} a & a \\ b & 0 \end{vmatrix} + \cancel{\begin{vmatrix} c & c & a \\ c & c & c \\ c & c & c \end{vmatrix}}$$

$$+ \begin{vmatrix} a+b & a+b & 0 \\ a+b & a+b & a+b \\ a+b & \cancel{a+b}\ 0 & a+b \end{vmatrix}$$

$$\begin{vmatrix} a & a & 0 \\ a & a & a \\ a & 0 & a \end{vmatrix} + \begin{vmatrix} b & b & 0 \\ b & b & b \\ b & 0 & b \end{vmatrix}$$

$$= -c \begin{vmatrix} a & a \\ b & 0 \end{vmatrix} + a^3 \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} + b^3 \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -c \begin{vmatrix} a & a \\ b & 0 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} (a^3+b^3)$$

$$= -c \begin{vmatrix} a & a \\ b & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} (a^3+b^3)$$

$$= -c \begin{vmatrix} a & a \\ b & 0 \end{vmatrix} + \Big\{ - \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} (a^3+b^3)$$

$$= -c(-ab) + (a^3+b^3)$$

$$= a^3 + abc + b^3.$$

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= \begin{vmatrix} b+c+b+c & a+c+a+c & a+b+a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 2b+2c & 2a+2c & 2a+2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= 2\left\{ \begin{vmatrix} b & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} + \begin{vmatrix} c & a & a \\ b & c+a & b \\ c & a & a+b \end{vmatrix} \right\}.$$

$$= 2\left\{ \begin{vmatrix} 0 & -a & 0 \\ b & c+a & b \\ c & c & a+b \end{vmatrix} + \begin{vmatrix} 0 & 0 & -b \\ b & c+a & b \\ c & a & a+b \end{vmatrix} \right\}.$$

$$= 2\left\{ a\begin{vmatrix} b & b \\ c & a+b \end{vmatrix} - b\begin{vmatrix} b & c+a \\ c & a \end{vmatrix}\right\}.$$

$$= 2\{a(b(a+b) - bc) - b(ba - c(c+a)\}$$

$$= 2\{a(ab + b^2 - bc) - b(ba - c^2 - ca)\}$$

$$= 2\{a^2b + \cancel{ab^2} - \cancel{abc} - \cancel{b^2a} + bc^2 + \cancel{bca}\}$$

$$= 2\{a^2b + bc^2\}.$$

$\underline{4abc}$

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\begin{vmatrix} a+b+c & a & a \\ a+b+c & c+a & b \\ 2c & c & a+b \end{vmatrix}$$

$C_1 - C_2$

$$\begin{vmatrix} a+b+c & 2a & a \\ a+b+c & a+b+c & b \\ 2c & a+b+c & a+b \end{vmatrix}$$

$C_2 - C_3$

$$\begin{vmatrix} -a+b+c & a & a \\ 0 & a+c+b & b \\ a+b-c & a+b+c & a+b \end{vmatrix}$$

27 i).

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \qquad \begin{matrix} R_1 - R_2 \\ R_2 - R_3 \end{matrix}$$

$$\begin{vmatrix} c & -a & a-b \\ b-c & a & -a \\ c & c & a+b \end{vmatrix} \qquad \begin{matrix} R_1 + R_2 \\ R_2 + R_3 \end{matrix}$$

$$\begin{vmatrix} b & 0 & -b \\ b & a+c & b \\ c & c & a+b \end{vmatrix} \qquad \begin{matrix} \cancel{R\ C_1 + C_2} \\ C_3 + C_1 \end{matrix}$$

$$\begin{vmatrix} b & 0 & 0 \\ b & a+c & 2b \\ c & c & a+b+c \end{vmatrix}$$

$$= b \begin{vmatrix} a+c & 2b \\ c & a+b+c \end{vmatrix}$$

$$= b \left\{ (a+c)(a+b+c) - 2bc \right\}$$

$$= b \left\{ a^2 + ab + ac + ac + cb + c^2 - 2bc \right\}$$

$$= b \left\{ a^2 + ab + 2ac - bc + c^2 \right\}.$$

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \begin{matrix} R_1 - R_2 \\ R_2 - R_3 \\ = \end{matrix} \begin{vmatrix} c & -c & a-b \\ b-c & a & -a \\ c & c & a+b \end{vmatrix}$$

$$\cancel{R_2 + R_3} =$$

$$R_1 + R_3 = \begin{vmatrix} 2c & 0 & aa \\ b-c & a & -a \\ c & c & a+b \end{vmatrix}$$

$$\cancel{C_1 - C_2 = \begin{vmatrix} 2c & 0 & a \\ b-c-a & a & -a \\ 0 & c & a+b \end{vmatrix}}$$

$$R_3 + R_2 = \begin{vmatrix} 2c & 0 & a \\ b-c & a & 0 \\ c & c & a+b+c \end{vmatrix}$$

$$R_1 - R_2 = \begin{vmatrix} 2c & 0 & a \\ b-c-a & a & 0 \\ 0 & c & a+b+c \end{vmatrix}$$

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = \begin{vmatrix} a+b+c & a & 2a \\ a+b+c & c+a & a+b+c \\ 2c & c & a+b+c \end{vmatrix}$$

$R_1 + R_2 \quad R_3 + R_2$

$$R_2 + R_1 \quad R_1 + R_3 = \begin{vmatrix} a+b+c & a+b+c & a \\ 2b & a+b+c & b \\ a+b+c & 2c & a+b \end{vmatrix}$$

ii). $$\begin{vmatrix} a & 2a+b & a+2b \\ a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \end{vmatrix}$$

$$= \cancel{\begin{vmatrix} a & a & a \\ a+b & a+2b & a+3b \\ & & \end{vmatrix}}$$

$$= \cancel{\begin{vmatrix} a & a & a \\ a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \end{vmatrix}} + \cancel{\begin{vmatrix} 1 & b & 2b \\ b & 2b & 3b \\ a+2b & a+3b & a+4b \end{vmatrix}}$$

$$\cancel{+ \begin{vmatrix} 1 & b & 2b \\ a & a & a \\ a+2b & a+3b & a+4b \end{vmatrix} + \begin{vmatrix} 1 & b & 2b \\ a & a & a \\ 2b & 3b & 4b \end{vmatrix}}$$

$=$

15. $x^n + x^{-n} - 2\cos n\theta = R$

$$Rx^n = x^{2n} + 1 - 2x^n \cos n\theta.$$

Let. $x^{2n} - 2x^n \cos n\theta + 1 = 0$

$$\therefore \quad x^n = \frac{\cancel{4}\ \overset{\cos n\theta}{2} \pm \sqrt{4\cos^2 n\theta - 4}}{2}$$

$$= \overset{\cos n\theta}{1} \pm \sin n\theta.$$

$$\therefore \quad x^n = \cos(n\theta + 2r\pi) \pm \sin(n\theta + 2r\pi).$$

$$x = \cos\left(\theta + \frac{2r\pi}{n}\right) \pm \sin\left(\theta + \frac{2r\pi}{n}\right).$$

$\therefore$ taking quad. factors.

$$x^{2n} - 2x^n \cos n\theta + 1 = (x^2 - 2x\cos(\theta) + 1)\,(x^2 - 2x\cos(\theta + 2\pi/n))\dots\,(x^2 - 2x\cos(\theta + \tfrac{2(n-1)\pi}{n})).$$

$$\therefore Rx^n = \frac{(x^2 - 2x\cos\theta + 1)}{x^n}\,(x^2 + 1 - 2x\cos(\theta + 2\pi/n))\dots\,(x^2 + 1 - 2x\cos(\theta + \tfrac{2(n-1)\pi}{n})).$$

$$Rx^{n-1} = (x - 2\cos\theta + x^{-1})\,(x + x^{-1} - 2\cos(\theta + 2\pi/n))\dots\,(x + x^{-1} - 2\cos(\theta + \tfrac{2(n-1)\pi}{n})).$$

Ex 9c.

1. $z = \cos\theta + i\sin\theta.$

i). $z^2 + \frac{1}{z^2} = \cos 2\theta + i\sin 2\theta + \cos 2\theta - i\sin 2\theta$

$= 2\cos 2\theta.$ ✓

ii). $z - \frac{1}{z} = \cancel{\cos\theta +} 2i\sin\theta$ ✓

iii). $z^6 + \frac{1}{z^6} = \cancel{2} \cos 6\theta. \quad \cancel{2i\sin 6\theta}$

iv). $z^4 - \frac{1}{z^4} = \cancel{2\cos 4\theta.} \quad 2i\sin 4\theta.$ ✓

v). $z^2 + 2z + \frac{1}{z^2} + 2/z = 2\cos 2\theta + 2\cos\theta$

$= 2(\cos 2\theta + \cos\theta)$

$= 4\cancel{2}\sin 3\theta/2 \sin\theta/2.$

vi). $z^3 - \frac{2}{z} + 1 + 2z + \frac{1}{z^3} = 4i\sin\theta + 2\cos 3\theta + 1.$ ✓

2. $z = \cos\theta + i\sin\theta$.

i). $2\cos 4\theta = z^4 + 1/z^4$

ii). $2i\sin 5\theta = z^5 - 1/z^5$.

iii). $\cos 7\theta = 1/2\,(z^7 + z^{-7})$.

iv). $\cos^2\theta = \cancel{z + z^{-1}} \quad \cancel{z z^*}$.

$\frac{1}{4}(z + z^{-1})^2$

v). $\sin 4\theta = -\frac{1}{4}(z - z^{-1})^2$

vi). $\sin^4\theta \cos^4\theta = -\frac{1}{8}(z - z^{-1})^2(z + z^{-1})^2$

3.

i). $\cos^3\theta$.

$\cos\theta = (z + z^{-1})$

1.

$\therefore \cos^3\theta = (z + z^{-1})^3$

$= (z + z^{-1})(z^2 + 2 + z^{-2})$.

$= z^3 + 2z + z^{-1} + z + 2z^{-1}$
$+ z^{-3}$.

$= (z^3 + z^{-3}) + 2(z + z^{-1}) + (z + z^{-1})$

$= \cos 3\theta + 3\cos\theta$.

$\cos^3\theta.$

$$z = \cos\theta + i\sin\theta.$$

$$z^3 = (\cos\theta + i\sin\theta)^3$$

$$= \cos 3\theta + i\sin 3\theta.$$

$$= (\cos\theta + i\sin\theta)(\cos^2\theta - 2i\sin\theta\cos\theta - \sin^2\theta)$$

$$= \cos^3\theta$$

---

ii). $\sin^4\theta.$

$$\sin\theta = \frac{1}{2i}(z - z^{-1})$$

$$\therefore \sin^4\theta = \left(\frac{1}{2i}\right)^4 (z - z^{-1})^4.$$

$$= \frac{1}{16}(z^2 + 2 + z^{-2})(z^2 + 2 + z^{-2})$$

$$= \frac{1}{16}\left\{\cancel{z^4} + \cancel{2z^2} + 1 + 2z^2 + 4 + \cancel{2z^{-2}} + 1 + 2z^{-2} + \cancel{z^{-4}}\right\}.$$

$$= \frac{1}{16}\left\{z^4 + z^{-4} + 2z^2 + 2z^{-2} + 2z^2 + 2z^{-2} + 5\right\}.$$

$$= \frac{1}{16}\left\{\cos 4\theta + 4\cos 2\theta + 5\right\}.$$

iv). $\cos^7\theta = z^{-7}(z+z^{-1})^7$ $\quad 1+(z^{-2})^7$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$\boxed{\begin{aligned} = z^7 &+ 6z^6z^{-1} + 15z^5z^{-2} \\ z^0 &+ 20z^4z^{-3} + 15z^3z^{-4} + 6z^2z^{-5} \\ &+ (zz^{-6}) \end{aligned}}$$

$^2\theta)$

$(z+z^{-1})^2 = (z^2) + 2 + (z^{-2})$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

5(7c 3d 21 1f.
2c 12d 24 1f.

1c 1f

1 $\boxed{3}$ $\frac{1}{3}f$ — 1d — 7c.
$\frac{1}{2}f$ — 1d — 2c

-2

$\therefore$ 1c $\rightarrow$ $\frac{1}{21}f$ $\rightarrow$ 1d (?)
1c $\rightarrow$ $\frac{1}{4}f$ $\rightarrow$ 1d

$7c \quad 3d \quad 1f.$

$2c \quad 12d \quad 1f.$

$\boxed{30}$

$7c \Rightarrow 3d \Rightarrow 1f$

$\therefore 1d \Rightarrow 7c \Rightarrow \frac{1}{3}f.$

$\therefore 1d \Rightarrow 1c \Rightarrow \frac{1}{21}f.$

$2c \Rightarrow 12d \Rightarrow 1f.$

$\therefore 2c \Rightarrow 1d \Rightarrow \frac{1}{12}f$

$\therefore$ [redacted]c $\Rightarrow 1d \Rightarrow \frac{1}{24}f \quad \frac{3}{2} \quad \frac{2}{3}$

$\boxed{\frac{21 \times 2}{3}} \quad \boxed{\frac{43}{3}} \quad \boxed{\frac{\cancel{24}^{8} \times 2}{\cancel{3}_{1}}} = \boxed{\begin{array}{r} 16 \\ 21 \\ \hline 37 \end{array}}$

$\boxed{\begin{array}{r} 24 \\ 16 \\ \hline 40 \end{array}}$

$$\sqrt{-1} = \sqrt{-1}$$

$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$$

$$\therefore \frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}}$$

$$\sqrt{-1}\sqrt{-1} = \sqrt{1}\sqrt{1}$$

$$\therefore -1 = 1$$

3x Ex 3c.

1. $\begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix}$

$a = b \quad \Delta = 0 \quad \therefore \quad (a-b)$ is a factor.

$$\begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix} = k(a-b)$$

$a = 2, \quad b = 1$

$$\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} = 2 - 4 = -2 = k(2-1) = k$$

$$\therefore \quad k = -1$$

$$\therefore \begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix} = -(a-b).$$

2. $\begin{vmatrix} 3x & x \\ 4y & 8y \end{vmatrix}$

1. $\begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix} = ab \begin{vmatrix} 1 & 1 \\ a & b \end{vmatrix}$

$$= ab\,(a-b).$$

$$\begin{vmatrix} 6 & 6 & 6 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix} \underset{\substack{\text{col3} \\ -\text{col1}}}{\overset{\substack{\text{col2} \\ -\text{col1}}}{=}} \begin{vmatrix} 6 & 0 & 0 \\ 2 & -1 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 6\,(2+1) = \underline{18}$$

2. $\begin{vmatrix} 3x & x \\ 4y & 8y \end{vmatrix} = xy \begin{vmatrix} 3 & 1 \\ 4 & 8 \end{vmatrix}$

$= 20xy.$

3. $\begin{vmatrix} a-b & a+b \\ 2a-2b & 3a+3b \end{vmatrix} = (a-b)(a+b) \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

$= (a-b)(a+b).$

4. $\begin{vmatrix} a^2-b^2 & 2(a+b)^2 \\ (a-b)^2 & a+b \end{vmatrix} = \begin{vmatrix} (a+b)(a-b) & 2(a+b)^2 \\ (a-b)(a-b) & a+b \end{vmatrix}$

$= (a-b)(a+b) \begin{vmatrix} a+b & 2(a+b) \\ a-b & 1 \end{vmatrix}$

$= (a+b)(a-b)(a+b) \begin{vmatrix} 1 & 2 \\ a-b & 1 \end{vmatrix}$

$= (a+b)(a-b)(a+b)(1-2a+2b).$

5. $\begin{vmatrix} x & 3y & 2z \\ 2x & y & 3z \\ 3x & 2y & z \end{vmatrix} = xyz \begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix}$ c2 − c3 Then col3 − col1

$\begin{vmatrix} 6 & 6 & 6 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 6$ x $-x$

$= xyz \begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & +1 \\ 3 & 1 & -2 \end{vmatrix}$

$= xyz \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & -3 \\ 3 & 3 & -5 \end{vmatrix} = xyz \begin{vmatrix} -1 & -3 \\ 3 & -5 \end{vmatrix}$

$$\cancel{= xyz \begin{vmatrix} -1 & -2 \\ 3 & -5 \end{vmatrix}}$$

$=$

6. $$\begin{vmatrix} 0 & 6b & 4c \\ 4a & 0 & 6c \\ 6a & 4b & 0 \end{vmatrix} = abc \begin{vmatrix} 0 & 6 & 4 \\ 4 & 0 & 6 \\ 6 & 4 & 0 \end{vmatrix}$$ Sum into R1

$$= 10abc \begin{vmatrix} 1 & 1 & 1 \\ 4 & 0 & 6 \\ 6 & 4 & 0 \end{vmatrix}$$ ~~R2~~ - C2 - C1, C3 - C1

$$= 10abc \begin{vmatrix} 1 & 0 & 0 \\ 4 & -6 & 2 \\ 6 & 4 & -6 \end{vmatrix}$$

$$= 10abc \begin{vmatrix} -6 & 2 \\ 4 & -6 \end{vmatrix}$$

$$= 10abc \{36 - 8\}$$

$$= 280abc$$

$x^{-x} = y$

$\log_e y = (-x)\log_e x$

$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{x} - \log_e x$

$\therefore \frac{dy}{dx} = -y(1 - \log_e x)$

$= -x^{-x}(1 - \log_e x)$

7. $\cancel{\begin{vmatrix} 0 & x^2 & x^2-y^2 \\ x & 0 & \end{vmatrix}}$

7. $\begin{vmatrix} 0 & x^2 & x^2-y^2 \\ x-y & y & x+y \\ y-x & x & 0 \end{vmatrix}$

$= \cancel{\begin{vmatrix} 0 & x^2 & (x+y)(x-y) \\ x-y & y & (x+y) \\ y-x & x & 0 \end{vmatrix}}$

Let $x = y$

$\begin{vmatrix} 0 & x^2 & 0 \\ 0 & x & 2x \\ 0 & x & 0 \end{vmatrix} = \begin{vmatrix} 2x & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\therefore$ $(x-y)$ is a factor.

$x = -y$

$\begin{vmatrix} 0 & y^2 & 0 \\ -2y & y & 0 \\ 0 & -y & 0 \end{vmatrix}$ if $y = 0$ $\Delta = 0$

$\therefore$ $(x+y)$ is a factor

$$\begin{vmatrix} 0 & x^2 & x^2-y^2 \\ x-y & y & x+y \\ y-x & x & 0 \end{vmatrix} \qquad C_1 + C_2$$

$$= \begin{vmatrix} x^2 & x^2 & x^2-y^2 \\ x & y & x+y \\ y & x & 0 \end{vmatrix}$$

if $x = y$ $\quad \Delta = 0$ $\quad \therefore (x-y)$ is a factor.

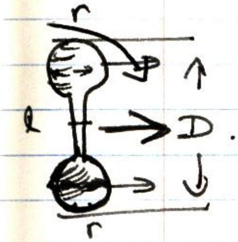
$$= (x+y) \begin{vmatrix} x^2 & x^2 & (x-y) \\ x & y & 1 \\ y & x & 0 \end{vmatrix}$$

$$\Delta = a_1A_1 + b_1B_1 + c_1C_1$$

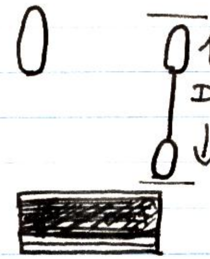
$$=$$

$$\cancel{P.E = \frac{Gmm}{r} = Fr}$$

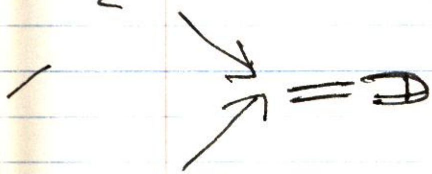
$$(x+y)\left|\begin{matrix} x^1 & x^2 & (x-y) \\ x & y & 1 \\ y & x & 0 \end{matrix}\right|$$

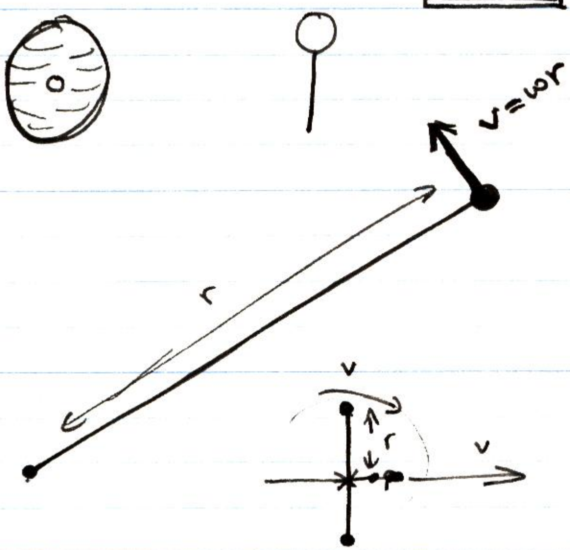


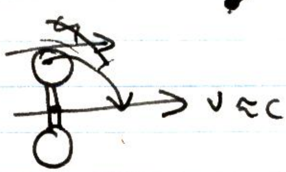
kr	km
kl	kl
kr	km



$v = 2at$







$\omega = \frac{v}{r}$

kr $\quad \omega = \frac{v}{kr}$

$$\begin{vmatrix} 0 & x^2 & x^2 - y^2 \\ x - y & y & x + y \\ y - x & x & 0 \end{vmatrix}$$

$\sin^3\theta \cos\theta$

z $\quad \sin\theta = \frac{1}{2i}(z - z^{-1}).$

$\quad \cos\theta = \frac{1}{2}(z + z^{-1}).$

1
1 1
1 2 1
1 3 3 1

$$\therefore \sin^3\theta = \frac{-1}{8i}\left(z(- z^{-1})\right)^3$$

$$= \frac{-1}{8i}\left(z^3 - 3z^2z^{-1} + 3zz^{-2} - z^{-3}\right).$$

$$= \frac{-1}{8i}\left(z^3 - 3z + 3z^{-1} - z^{-3}\right).$$

$$= \frac{-1}{8i}\left(\sin 3\theta - 3\cancel{\sin}\,\cancel{\cos}\sin\theta\right).$$

$$= \frac{-1}{8i}\cos\theta\left(\sin 3\theta - 3\sin\theta\right).$$

$\cos\theta + i\sin\theta - \cos\theta + i\sin\theta$

$= 2i\sin\theta$

$$\sin\theta = \frac{1}{2i}(z - z^{-1})$$

$$\sin^3\theta = -\frac{1}{8i}(z - z^{-1})^3$$

$$= \frac{-1}{8i}(z^3 - 3z + 3z^{-1} - z^{-3})$$

$$= \frac{-1}{8i}(z^3 - z^{-3} - 3(z - z^{-1})).$$

$$= \frac{-1}{8\cancel{i}}(2\cancel{i}\sin 3\theta - 6\cancel{i}\sin\theta).$$

$$= \frac{-1}{\cancel{8}_{4}}(\cancel{2}\sin 3\theta - \cancel{6}^{3}\sin\theta)$$

$$= -\frac{1}{4}(\sin 3\theta - 3\sin\theta)$$

$$= -\frac{1}{4}\cos\theta(\sin 3\theta - 3\sin\theta).$$

$A + B = 6\theta$

$$= -\frac{1}{4}(\cos\theta\sin 3\theta - 3\sin\theta\cos\theta)$$

$A - B = 2\theta$

$A = 6\theta - B$

$$= -\frac{1}{4}\left(\frac{1}{2}(\sin 2\theta + \sin 4\theta) - \tfrac{3}{2}\sin 2\theta\right)$$

$A = 2\theta + B$

$6\theta - B = 2\theta + B$

$$= -\frac{1}{8}(\sin 2\theta - 3\sin 2\theta + \sin 4\theta)$$

$2B = 4\theta$

$B = 2\theta$

$$= -\frac{1}{8}(\sin 4\theta - \sin 2\theta).$$

$0 \to 1$
$1 \to 1 \quad 1$
$2 \to 1 \quad 2 \quad 1$
$3 \to 1 \quad 3 \quad 3 \quad 1$
$4 \to 1 \quad 4 \quad 6 \quad 4 \quad 1$

$\cos^4\theta \sin^2\theta$

$$\cos\theta = \frac{1}{2}(z + z^{-1})$$

$$\sin\theta = \frac{1}{2i}(z - z^{-1})$$

$$\cos^4\theta = \frac{1}{16}(z + z^{-1})^4$$

$$= \frac{1}{16}\left\{z^4 + 4z^3z^{-1} + 6z^2z^{-2} + 4zz^{-3} + z^{-4}\right\}.$$

$$= \frac{1}{16}\left\{z^4 + z^{-4} + 4z^2 + 6 + 4z^{-2}\right\}.$$

$$= \frac{1}{16}\left\{2\cos 4\theta + 4(2\cos 2\theta) + 6\right\}.$$

$$= \frac{1}{16}\left\{2\cos 4\theta + 8\cos 2\theta + 6\right\}.$$

4. $2^6 \sin^5\theta \cos^2\theta = \sin 7\theta - 3\sin 5\theta + \sin 3\theta + 5\sin\theta .$

$\sin\theta = \frac{1}{2i}(z - z^{-1}).$

$\cos\theta = \frac{1}{2}(z + z^{-1}).$

$0 \rightarrow 1$
$1 \rightarrow 1 \quad 1$
$2 \rightarrow 1 \quad 2 \quad 1$
$3 \rightarrow 1 \quad 3 \quad 3 \quad 1$
$4 \rightarrow 1 \quad 4 \quad 6 \quad 4 \quad 1$
$5 \rightarrow 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$$\therefore \sin^5\theta = \frac{1}{2^5 i}(z - z^{-1})^5$$

$$= \frac{1}{2^5 i}\left(z^5 - 5z^4z^{-1} + 10z^3z^{-2} - 10z^2z^{-3} + 5zz^{-4} - z^{-5}\right).$$

$$= \frac{1}{2^5 i}\left(z^5 - z^{-5} - 5z^3 + 5z^{-3} + 10z - 10z^{-1}\right)$$

$$= \frac{1}{2^5 i}\left(2i\sin 5\theta - \cancel{5}\,10i\sin 3\theta + 20i\sin\theta\right)$$

$$= \frac{\cancel{2i}}{2^4\cancel{2}\cancel{i}}\left(\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\therefore \sin^5\theta\cos^2\theta = \frac{1}{2^5}(1 + \cos 2\theta)(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$$

$$= \frac{1}{2^5}\left(\sin 5\theta - 5\sin 3\theta + 10\sin\theta + \cos 2\theta\sin 5\theta - 5\sin 3\theta\cos 2\theta + 10\sin\theta\cos 2\theta\right).$$

$$= \frac{1}{2^5}\Big(\sin 5\theta - 5\sin 3\theta + 10\sin\theta + \cos 2\theta \sin 5\theta$$

$$+ 10\cos 2\theta \sin\theta - 5\sin 3\theta \cos 2\theta\Big).$$

$A+B = 10\theta$

$A-B = 4\theta$

$B = 10\theta - A = A - 4\theta$

$2A = 6\theta$

$\therefore A = 3\theta.$

$\therefore B = 7\theta$

$P+Q = 4\theta$

$P-Q = 2\theta$

$\cancel{Q = 4\theta - P}$

$P = 4\theta - Q = 2\theta + Q$

$\therefore 2Q = 2\theta$

$\therefore Q = \theta. \quad \therefore P = 3\theta$

$E+F = 6\theta$

$E-F = 4\theta$

$E = 6\theta - F = 4\theta + F$

$\therefore 2F = 2\theta$

$\therefore F = \theta$

$\therefore E = 5\theta$

$$= \frac{1}{2^5}\Big(\sin 5\theta - 5\sin 3\theta + 10\sin\theta + \tfrac{1}{2}(\sin 3\theta + \sin 7\theta)$$
$$+ 5(\sin 3\theta - \sin \theta) - \tfrac{5}{2}(\sin\theta + \sin 5\theta)\Big).$$

$\times\ 2^6$

$$\cancel{2\sin 5\theta - 10\sin 3\theta + 20\sin\theta + \sin 3\theta + \sin 7\theta}$$
$$\cancel{+ 5\sin 4\theta - 5\sin 2\theta - 5\sin\theta - 5\sin 5\theta.}$$

$$\cancel{= -3\sin 5\theta - 9\sin 3\theta - 5\sin 2\theta + \sin 7\theta + 5\sin 4\theta}$$
$$\cancel{+ 15\sin\theta.}$$

$$= \cancel{2\sin 5\theta} \cancel{- 10\sin 3\theta} + 20\sin\theta + \cancel{\sin 3\theta} + \cancel{\sin 7\theta}$$
$$\cancel{+ 10\sin 3\theta} - 10\sin\theta - 5\sin\theta \cancel{- 5\sin 5\theta}.$$

$$= \sin 7\theta - 3\sin 5\theta + \sin 3\theta + 5\sin 3\theta.$$

---

5. i). $\cos 4\theta$

$$\begin{array}{l} 0 \to 1 \\ 1 \to 1\ 1 \\ 2 \to 1\ 2\ 1 \\ 3 \to 1\ 3\ 3\ 1 \\ 4 \to 1\ 4\ 6\ 4\ 1 \end{array}$$

$$z = \cancel{\sin\theta +}\ \cos\theta + i\sin\theta$$
$$z^4 = \cos 4\theta + i\sin 4\theta$$
$$= (\cos\theta + i\sin\theta)^4$$
$$= \cos^4\theta + 4\cos^3\theta(i\sin\theta) + 6\cos^2\theta(i\sin\theta)^2$$
$$+ 4\cos\theta(i\sin\theta)^3$$
$$+ (i\sin\theta)^4$$

$\Re$
$$= \cos^4\theta + \cancel{4\cos^3} \ 6\cos^2\theta(i\sin\theta)^2 + (i\sin\theta)^4$$
$$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$$
$$= \cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$$
$$= \cos^4\theta - 6\cos^2\theta + 6\cos^4\theta + (1 - 2\cos^2\theta + \cos^4\theta).$$
$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$

ii). $\dfrac{\sin 4\theta}{\sin\theta}.$

$$z = \cos\theta + i\sin\theta$$
$$z^4 = (\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta.$$

g^* $\sin 4\theta = 4\cos^3\theta(i\sin\theta) + 4\cos\theta(i\sin\theta)^3$

$$\frac{\sin 4\theta}{\sin\theta} = 4\cos^3\theta \cancel{\sin\theta} - 4\cos\theta \cancel{\sin^3\theta}.\sin^2\theta.$$
$$= 4\cos^3\theta - 4\cos\theta(1-\cos^2\theta).$$
$$= 4\cos^3\theta - 4\cos\theta + 4\cos^3\theta$$
$$= 8\cos^3\theta - 4\cos\theta.$$

iii). $\cos 6\theta$

$$z = \cos\theta + i\sin\theta$$
$$\therefore z^6 = \cos 6\theta + i\sin 6\theta = (\cos\theta + i\sin\theta)^6$$

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
~~1 6 15 20 15~~
1 6 15

$$= \cancel{\cos^6\theta + 5\cos^5\theta(i\sin\theta) + 10\cos^4\theta(i\sin\theta)^2 + 10\cos^3\theta(i\sin\theta)^3 + 5\cos^2\theta(i\sin\theta)^4}$$

$$= \cos^6\theta + {}^6C_5\cos^5\theta(i\sin\theta)$$
$$+ {}^6C_4\cos^4\theta(i\sin\theta)^2$$
$$+ {}^6C_3\cos^3\theta(i\sin\theta)^3$$
$$+ {}^6C_2\cos^2\theta(i\sin\theta)^4$$
$$+ {}^6C_1\cos\theta(i\sin\theta)^5$$

$$^nC_r = \frac{n!}{r!\,(\cancel{n-r}^{\,6})!}$$

$$+ \; ^6C_0 \; \cancel{\cos\theta} \; (i\sin\theta)^6$$

$$= \frac{6!}{1!\,(5!)} = \cancel{6}/$$

$$\Re \quad \cos 6\theta = \cos^6\theta \; - \; ^6C_4 \cos^4\theta \sin^2\theta$$

$$+ \; ^6C_2 \cos^2\theta \sin^4\theta$$

$$\frac{6!}{4! \; 2!}$$

$$= \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta$$

$$= \cos^6\theta - 15\cos^4\theta(1-\cos^2\theta) + 15\cos^2\theta(1-\cos^2\theta)^2$$

$$= \frac{5.\cancel{6}3}{1.\cancel{2}}$$

$$= \cancel{\cos^6\theta} - 15\cos^4\theta + \cancel{15\cos^6\theta}$$

$$+ 15\cos^2\theta(1 - 2\cos^2\theta + \cos^4\theta).$$

$$\frac{6}{2! \; 4!}$$

$$= 16\cos^6\theta - 15\cos^4\theta + 15\cos^2\theta - 30\cos^4\theta$$

$$+ 15\cos^6\theta.$$

$$\begin{array}{r} {}^{1}16 \\ 15 \\ \hline 31 \end{array}$$

$$= 31\cos^6\theta - 45\cos^4\theta + 15\cos^2\theta + 1$$

7.

i). $\text{Tan } 3\theta = \dfrac{\sin 3\theta}{\cos 3\theta}$.

$$\begin{array}{cccc} & & 1 & 1 \leftarrow 0 \\ & 1 & 1 & 1\leftarrow 1 \\ 1 & 2 & 1 & 1 \leftarrow 1 \\ 1 & 3 & 3 & 1 \leftarrow 3 \end{array}$$

~~$\sin\theta =$~~

$$z = \text{cis } \theta$$

$$z^3 = \text{cis } 3\theta = (\cos\theta + i\sin\theta)^3$$

$$= \cos^3\theta + 3\cos^2\theta\, i\sin\theta$$

$$+ 3\cos\theta\, i^2\sin^2\theta$$

$$+ (i\sin\theta)^3$$

$\Re$ $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$.

$\Im$ $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$.

$$\therefore \text{Tan } 3\theta = \frac{3\cos^2\theta\sin\theta - \sin^3\theta}{\cos^3\theta - 3\cos\theta\sin^2\theta}.$$

$$= \frac{3\text{Tan }\theta - \text{Tan}^3\theta}{1 - 3\text{Tan}^2\theta}.$$

$I = \int x^2 \sin x \, dx$ $\qquad u = x^2$ $\qquad \int x^2 \sin x \, dx$

$du = 2x\,dx \qquad dv = \sin x \, dx$

$v = -\cos x$

$I = -x^2 \cos x + \int \cos x \; 2x \, dx$

8. $$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$\theta = 0, \ \tfrac{1}{3}\pi, \ \tfrac{2}{3}\pi.$

$\tan\theta = 0 \quad \therefore \quad \tan 3\theta = 0,$

$$\tan 3\theta = 0 = \frac{3t - t^3}{1 - 3t^2}$$

$\therefore \quad 3t - t^3 = 0$

$\therefore \quad t(3 - t^2) = 0$

$\therefore \quad t = 0 \quad \text{or} \quad 3 - t^2 = 0$

$\therefore \quad t^2 = 3$

$\therefore \quad t = \pm\sqrt{3}$

$\cancel{1 + z\cos\theta}$

$\cancel{(1 + (\cos\theta + i\sin\theta)\cos\theta)}^{-1}$

$z = \cos\theta + i\sin\theta.$

$z^5 = \cos 5\theta + i\sin 5\theta$

$= \overset{\cos^6\theta}{1} + {}^6C_5\cos^5\theta(i\sin\theta) + {}^6C_4\cos^4\theta(i\sin\theta)^2$
$+ {}^6C_3\cos^3\theta(i\sin\theta)^3 + {}^6C_2\cos^2\theta(i\sin\theta)^4$
$+ {}^6C_1\cos^1\theta(i\sin\theta)^5$
$+ {}^6C_0\,\cancel{\cos}\,(i\sin\theta)^6.$

$\Re\ \cos 5\theta = \cos^6\theta - \cancel{{}^6C_4\cos^4\theta\,\sin}$
$- {}^6C_4\cos^4\theta\sin^2\theta$
$+ {}^6C_2\cos^2\theta\sin^4\theta$
$\cancel{+} - {}^6C_0\sin^6\theta.$

$= \cos^6\theta -$

Stamp Club Report.

Mention J. Allday - President
(Mascot) R. James - who actually did the work. 1st day covers

~~X~~mas Fair Stamp Sale raised $\approx$ £12

Approvals service

Catalogues (out of date)

Apathy

$i^6 = (\sqrt{-1})^6 = -x^3 = -$

$\dfrac{6!}{\cancel{4!}\,2!} \dfrac{5.6}{1.2} = 15$

$\cancel{-1}$

$\cancel{i \times i} \times i \times \cancel{i \times i} \times i \times i$

$\cancel{-1}$ $\quad -1 \quad -1$

$= -1$

	MAC.	CASE.	SMITH.	KNEALE	HUGHES.	CALLAGHA	JONES.	DALGLISH.	HEIGHWA	KENNEDY	CLEMENC
6											
2 CHICKEN											
2 Fish											
1 omelettes	-	-	-	✓	-	-	-	-	-	-	-
4 potatoes				·	✓	✓	✓				•
7 veg.					✓	✓	✓				

$$I_n = \int_0^1 x^n e^{-x}\,dx \qquad (>0)$$

$u = e^{-x}$ $\qquad dv = x^n\,dx$

$du = -e^{-x}\,dx$ $\qquad v = \frac{x^{n+1}}{n+1}$

$du = e^{-x}\,dx$

$u = -e^{-x}$ $\qquad v = x^n$

$\qquad\qquad dv = n x^{n-1}\,dx$

uv

$$I_n = \int -e^{-x}\cdot x^n + \int n x^{n-1} e^{-x}\,dx$$

$$= -\frac{x^n}{e^x} + n\int_0^1 x^{n-1} e^{-x}\,dx$$

$$= \left[-\frac{x^n}{e^x}\right]_0^1 + \left[n\,I_{n-1}\right]_0^1$$

$$= -\frac{1}{e} + n\,I_{n-1} \qquad n>0$$

$$\sqrt{-1} = \sqrt{-1}$$

$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$$

$$\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}}$$

$$\sqrt{-1} . \sqrt{-1} = \sqrt{1} . \sqrt{1}$$

$$\therefore \quad -1 = 1$$